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Different Aspects of Evolutionary Algorithms, Multi-Objective Optimization Algorithms and Application Domain

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ABSTRACT

This paper presents the basic introduction of the Optimization, Multi-Objective Optimization algorithms, Multi-Objective Optimization Problems and its application domain. It also emphasizes the differences and needs of the optimization on the basis of a single objective and multi objective criteria. It explains various techniques to evaluate the Multi-Objective Optimization Problems. We present the gracefulness of Genetic Algorithms for solving the problems through Multi-Objective Genetic Algorithms and also compare various methods. We also represent the difficulties came in to consideration to solve a Multi-Objective Optimization Problem.

Keywords - **Evolutionary Program, Evolutionary Strategy, Genetic Algorithm, Multi-Objective Optimization Problems, Optimization, Single Objective Optimization Problems .**

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I. INTRODUCTION

Optimizing is a procedure of finding and comparing feasible solutions until no better solutions can be found. Like if we are talking about the software, the software should be best if its cost is minimum, output/throughput is maximum, for PCB designing, its cost of fabrication is minimum and product reliability is maximum. When an optimization problem involves only one objective function, the task of finding the optimal solution is called Single Objective Optimization. On the other hand, when more than one objective function, the task of finding one or more optimum solutions is known as Multi-Objective Optimization. Evolutionary Algorithms are search methods based on the natural solution and survival of the fittest in the biological world. Evolutionary Algorithm differ from other Optimization technique in that they involve search from "Population" of solutions. Evolutionary Algorithms (EAs) can be used to find multiple Optimal solutions in one single simulation run (due to their population approach). So Evolutionary Algorithms is used for solving Multi-Objective Optimization Problems (MOOPs).

I.I APPLICATION DOMAIN

There are four application domains namely Evolutionary Strategy (ES), Evolutionary Programming (EP), Genetic Programming (GP) and Genetic Algorithm (GA). Evolutionary Strategy optimize continuous functions with recombination. Evolutionary Programming was introduced by Lawrence J. Foget [12] in 1960. He gave an idea of simulated learning process to generate Artificial Intelligence. Evolutionary Program optimizes continuous functions without recombination. Genetic Programming involves programming and the Genetic Algorithm is used to optimize general combinatorial problems and used in artificial intelligence to find optimized solutions for example in Engineering Design to optimize structural design of buildings and machines, in Robotics applications, in Image Processing to find more enhanced version of image [10][11]. Genetic Algorithms (GAs) [7] applies the principles of evolutions found in the nature. In GAs, the problem is encoded in a series of bit strings that are manipulated by the algorithm. In other words, GAs is a stochastic search method inspired by the evolution and adaptation in biological systems. It is first presented by John Holland in 1975. The search is conducted directly in the solution space, and each solution is encoded in a certain way and is called an individual. The population of individuals is maintained by its fitness function or quality of individuals. The population is improved by crossover, recombination of genetic material from different individuals. Genetic Algorithm mainly consists of five points they are :

1. A Chromosomal representation of solution,
2. Population,
3. Fitness function,
4. Genetic operators and Parameters of Genetic Algorithm.

To formulate the process of natural solution in a computer, a

method is needed to encode potential solution to that problem in a form that a computer can process is known as representation technique of chromosome.
For example binary representation of chromosomes having value 5.0 and 3.0 can be represented as :

0	1	0	1

Chromosome1

0	0	1	1

Chromosome2

Population is a group of individuals, which may interact to each other, for example by mating, producing offspring. Basically it is a set of solutions. The role of the fitness function is to evaluate individuals and assign them a fitness value. Fitness means adaptness. The fit are those who fit their existing environments and whose descendants will fit future environments.
Selection crossover and mutation are the Genetic Operators which provides the functionality of the Genetic Algorithm. Selection mechanism, provides a methods to select individuals which are to be copied over to the next generation. It is a process of deciding which individuals should be allowed to contribute to the next generation with their genetic material like elitist selection, Roulette-Wheel Selection, Scaling Selection, Tournament Selection etc. After selection, Crossover operation is used to exchange the genetic material to form the children for the next generation. In Crossover, two individuals are chosen to swap segments of their code, to produce offsprings or childrens. It can be of many types; Single Point crossover, Two Point crossover, Uniform Crossover and Arithmetic Crossover :

Example 1 (for One Crossover Point and Two Crossover Point)

	One Crossover Point	Two Crossover Point
Parent 1	1111 | 111111	1111 | 111 | 111
Parent 2	0000 | 000000	0000 | 000 | 000
Offspring 1	1111 | 000000	1111 | 000 | 111
Offspring 2	0000 | 111111	0000 | 111 | 000

Example 2 (for Arithmetic Crossover)

In Arithmetic Crossover, some arithmetic operation is performed to make a new offspring and it can be defined as a linear combination of two chromosomes such as :

c1=a*x+(1-a)*y and
c2=(1-a)*x+a*y

Where :

c1 and c2 are offspring or child1 and child 2 respectively,
x and y be two parents in the mating pool,
and a is a random number where a Є [0,1].

Basic criteria to improve the performance of Genetic Algorithm are defined in terms of population size, crossover rate and mutation rate. Population may be large or small. If there is a large population then, it does not mean that it may improve the performance of GA with regard to the speed of finding the solution. Generally population size may be chosen in between 50-100. But the best population size depends on the encoding of the string or chromosome.
For the crossover rate, it should be high but the mutation rate, it should be chosen very low .5% to 1% for good performance of Genetic Algorithm.
Overall Genetic Algorithm is very easy and powerful algorithm for solving a wide range of problems. The general steps of Genetic Algorithms may be expressed as :

Step I : Initially generate N numbers of solutions in a random fashion. These solutions are knows as first population P_o .
Step II : Evaluate fitness in $P_{o.}$
Step III : Apply crossover to generate offspring population P_c .
Step IV : Apply mutation for each solutions, e.g. if x is a solution then x Є $P_{c.}$
Step V : Evaluate fitness value for each solutions x Є P_c , based upon some objective criteria.
Step VI : Select N number of solutions from P_c , based on their fitness value and set as $P_{c+1.}$
Step VII : Terminate searching mechanism and return the current population if the convergence criteria is satisfied, else
set time t=t+1 and
move to step III.

I.II DIFFERENCES

Most striking difference to classical search and optimization algorithms is that Evolutionary Algorithms use a population of solutions in each iteration, instead of a single solution. Since a population of solutions are processed in each iteration, the outcome of an EAs is also a population of solutions.
The ability of an Evolutionary Algorithm is to find multiple optimal solutions in one single simulation run makes Evolutionary Algorithms unique in solving Multi-Objective Optimization Problems (EAs uses a population of solutions in each iteration and a classical method uses only one solution That's why we call EAs as a population based approach and Classical method a Point to Point approach).

I.III MULTI-OBJECTIVE OPTIMIZATION PROBLEM

Genetic Algorithm are the most popular heuristic technique to solve Multi-Objective Design and Optimization problems [8]. Most real world problems have multiple objectives to achieve, this situation creates a set of problems in Operation Research (OR) called Multi-Objective Optimization Problems. A Multi-Objective Optimization problem has a number of objective functions, which are to be minimized or maximized. The general form Multi-Objective Optimization problems(MOOP) can be expressed as :

General Form of Multi-Objective Optimization Problem

Minimize/Maximize $f_m(x)$, $m=1,2,\ldots\ldots M$;
Subject to $g_j(x)>=0$, $j=1,2,\ldots\ldots J$;
$x_i^{(L)} <= x_i <= x_i^{(U)}$ $i=1,2,\ldots\ldots n$;

where :

n - number of decision variable,

$x_i^{(L)}$, $x_i^{(U)}$ - lower and upper bounds of solution i,
J - number of inequality constraint functions,
M - number of objective functions

Multi-Objective Optimization is sometimes referred as vector optimization, because a vector of objectives, instead of a single objective, is optimized. MOOP can be of many types :

1. **Linear MOOP**
2. **NonLinear MOOP**
3. **Convex MOOP**
4. **NonConvex MOOP**

For a Linear MOOP(Multi-Objective Optimization Problem), if all the objective functions are linear, the resulting MOOP is called a Multi-Objective Linear Program(MOLP). The name Linear Optimization comes from the fact that the quantity which is to be optimized is linear function of the unknown quantity and the constraints. For a NonLinear MOOP, if all the objective functions are non linear, the resulting MOOP is called a Non Linear Multi-Objective Problem. For Non Linear problems, the solution techniques often donot have convergence proof. And for a Convex MOOP, if all the objective functions are convex and the feasible region is convex. For example the optimization problem.

Minimize f(x) such that $x \in F$
Is called convex if
(i) the feasible region $F \subset R^n$ is convex.
(ii) the objective function $f : F \longrightarrow R$ is convex.

I.IV MULTI-OBJECTIVE OPTIMIZATION ALGORITHMS

As in Single-Objective Optimization Problem only single optimal solution is required but for Multi-Objective Optimization Problem, it produces a set of solutions which are superior to the rest of the solutions with respect to all objective criteria but are inferior to other solutions in one or more objectives. These solutions are called Pareto Optimal solutions or non-dominated solutions.

The Primary goal of Multi-objective Optimization Algorithm is to identify solutions in the Pareto optimal set. Since the size of Pareto Optimal set is infeasible so to find out entire Pareto Optimal set is practically not possible. Therefore, in a practical point of view, we find the best set of solutions that represents the Pareto Optimal set. Commonly, there are two approaches, they are :

1. Ideal Approach
2. Preference Based Approach

In Ideal approach, no special importance is given to any particular objective and a set of trade off or Pareto Optimal solutions are desired to be found. After a set of Pareto Optimal solutions (or near to Pareto Optimal solution) is found, some higher-level information is needed regarding the problem for choosing one solution from the obtained set of solutions.Evolutionary Multi-Objective Optimization Algorithm follows this approach. In Ideal approach of Multi-Objective Optimization, two tasks must do well, they are :

a) **Converge as close to the true Pareto Optimal solutions as possible.**
b) **Maintain as diverse a population as possible.**

In most MOEAs, convergence towards the Pareto Optimal front is achieving by assigning a fitness based on the non domination ranking of solution. Diversity among solutions is achieved by using an explicit niching or crowding operation.

In Preference Based approach, Instead of finding a set of Pareto Optimal solutions, the focus is to find one of the Pareto Optimal solution based on a user-specified relative importance vector for the objectives. Classical Multi-Objective Optimization Algorithms follows this approach. In Classical Multi-Objective Optimization there exist no studies related to non-dominated sorting.

I.V BASIC IDEA BEHIND MULTI-OBJECTIVE OPTIMIZATION

In a general Multi-objective optimization problem, a function f is such that it maps $f : X \longrightarrow Y$

Where X is a decision space and x1,x2…………xn $\in$ X
here x1,x2………xn are decision vectors.
And Y is a Objective space and y1,y2,…………..yn $\in$ Y
here y1,y2……....yk (for k objectives) are objective vectors.
Let $Y \subseteq R^k$ (Objective space is a subset of Real Numbers) and k>1 (means Multi-Objective)

Since there is a difficulty to compare two solutions x1 & x2 so Pareto Optimal concept came in existence.

x1 dominates x2 means x1>x2 if f(x1) dominates f(x2).

y1 dominates y2 i.e. y1>y2, if no component of y1 is smaller than the corresponding component of y2 and at-least one component is greater.

Hence we can find Optimal solutions, i.e. solutions non-dominated by any other solution. Hence there may exist several Optimal objective vectors representing different trade-offs between the objectives.

II. SEVERAL WAYS TO APPROACH A MULTI-OBJECTIVE OPTIMIZATION PROBLEM

There are many ways to solve a Multi-Objective Optimization Problems, they are :

1. Function Aggregation or Weighted Sum Approach.
   Population Based Approach.
2. Pareto Based Approaches.
3.1 Fonesca & Fleming MOGA (FFGA)
3.2 Niched Pareto Genetic Algorithm (NPGA)
3.3 Non-dominated Sorting Genetic Algorithm (NSGA).

3.4 Non Dominated Sorting Genetic Algorithm II (NSGA II)
3.5 Strength Pareto Evolutionary Algorithm (SPEA)
3.6 Strength Pareto Evolutionary Algorithm II (SPEA II)

II.1 Function Aggregation (Weighted Sum Approach)

It is a very simple and well known approach. In this approach each criteria is assigned a weighted value. In this (Weighted Sum) Approach, objectives are combined in a higher scalar function and used to calculate fitness.

e.g. $$\min \sum_{i=1}^{k} w_i f_i(x)$$

where

$f_i(x)$ is normalized objective function,

$w_i \geq 0$ are weighting coefficients representing, and k → objectives and assumed that

$$\sum_{i=1}^{k} w_i = 1$$

The functions can be linear or nonlinear. The weighted sum is especially used in cases when the varying significance of the individual criteria is known or can be estimated. For a given weight vector w = {w1, w2,…..wk} yields a single solution, but if multiple solutions are desired, the problem must be solved multiple times with different weight combinations. The main difficulty with this approach is selecting a weight vector for each run. The main advantage of this approach is this that it is very simple to implement.

II.2 Population Based Approach (Non Pareto Approach)

In Population Based Approach, this approach is able to produce multiple non-dominated solutions concurrently in a single simulation run e.g. VEGA (Vector Evaluated genetic Algorithm) by Schaffar[1], Aggregating by variable objective weighting etc.

II.3 Pareto Based Approach

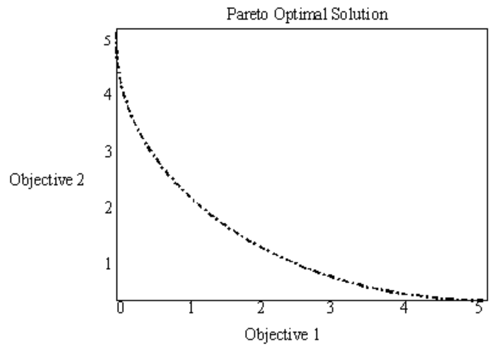
In Pareto Based Approach, this approach gives the concept of Pareto Optimality in their selection mechanism. e.g. Niched Pareto Genetic Algorithm[2], Strength Pareto Genetic Algorithm[6] etc. In this approach solutions are ranked based on the dominance relation **(*While in the Population Based Approach, the solutions are ranked for each objective function*)**.

Definition I : A solution →v is Pareto Optimal if there is no other vector u in the search space that dominates v.

Definition II: The solutions which are Non Dominated Solutions, are called Pareto Optimal Solutions.

II.3.1 Fonesca & Fleming MOGA (FFGA)

It was proposed by Fonesca et. al. [3] which is a ranking based approach. Each individual in the population is ranked based on how many other points dominate them. All Non-dominated Individuals are assigned the same rank and obtain same fitness, means they all have the same probability of being selected. Unlike single-objective optimization, the solution to a multi-objective problem is a family of points or collection of points known as the Pareto-Optimal set.



II.3.2 Niched Pareto Genetic Algorithm (NPGA)

It was proposed by Goldberg, D.E. et. al. [2].This is an interesting form of tournament selection called Pareto domination tournaments.
In this scheme two members of the population are chosen randomly and they are each compared to a subset of the population. If one is non-dominated and other is not, then the non-dominated one is selected. If there is a tie (means both are either dominated or non-dominated), then fitness sharing decides the tourney results. The advantages of the Algorithm is that there is no restriction on the number of optimized objectives. This Algorithm has a diversity-preserving mechanism to overcome the premature convergence problem.

II.3.3 Non Dominated Sorting Genetic Algorithm (NSGA)

This is developed by Srinivas and Deb[4]. In this approach before selection is performed, the population is ranked on the basis of non-domination means all non-dominated individuals are given a fitness value (dummy) which is directly proportional to the population size; for dominating the density of the problem, these individuals are shared with the assigned dummy fitness value. Now these individuals are ignored & another layer of non-dominated individuals is considered. This process continues until all individuals in the population are classified. In this technique stochastic remainder proportionate selection is used for fitness assignment.

II.3.4 Non Dominated Sorting Genetic Algorithm II

This is fast non-dominated sorting Genetic Algorithm and was proposed by Deb. K. et. al. [5] and uses Elitism. While creating a new population by crossover and mutation, there can be a big chance that some of the best individuals may lost. Elitism is a mechanism, that first copies the best

individuals or few best individuals to the new population. Hence Elitism prevents a loss of the best found solutions. Elitism can rapidly increase the performance of Genetic Algorithm and also increase the convergence speed of the Genetic Algorithm.
In Non Dominated Sporting Genetic Algorithm II (NSGA II), a crowded comparison operator is used that ranks the population based on both Pareto dominance & region density. Crowded Operator plays important role to fast-up this Algorithm. Mainly three strategies are considered for this Algorithm;

1. **This Algorithm use Elitism strategy.**
2. **This Algorithm use explicit diversity preserving strategies.**
3. **It emphasizes Non Dominated Solutions.**

II.3.5 Strength Pareto Evolutionary Algorithm (SPEA)

This algorithm is Proposed by Zitzler & Thiele [6]. This is a Recent technique for finding or approximating the pareto set for multi-objective optimization problem. This technique finds multiple pareto optimal solution in parallel. It stores the nominated solution found so for externally.

- It uses concept of Pareto dominance in order to assign scalar fitness values to individuals.
- It performs clustering to reduce the number of non-dominated solutions stores without destroying the characteristics of the trade-off front.

Strength Pareto Evolutionary Algorithm is different because:

1. ***The fitness of an individual is determined only from the solutions stored in the external non-dominated set. Whether members of the population dominate each other is irrelevant.***
2. ***All solutions in the external non-dominated set participate in the selection.***
3. ***A new niching method is provided in order to preserve in the population; this method is Pareto based and does not require any distance parameter (like the niche radius for sharing).***

II.3.6 Strength Pareto Evolutionary Algorithm II

This algorithm is an improved version of SPEA. It adjusts slightly the fitness strategy and clustering. In addition archiving mechanism enhancements allow for the presentation of boundary solution that are missed with SPEA.

Difficulties find with Multi-Objective Optimization Problem:

A number of problems may be arises if the problem is solved through Multi-Objective Genetic Algorithm, they can be explained as :

1. ***Search space of the problems depends on a large number of decision variables or parameters, called large parameter optimization problem (LPOPs).***
2. ***It produces difficulties in locating multiple Optimal peak of solutions (because Pareto Optimal solutions may be similar to the other Pareto Optimal solutions for some variables).***
3. ***If we increase the objectives automatically the number of Pareto Optimal solutions increases. They may be(Pareto Optimal solution) infinite for continuous parameters problems. So it creates difficulties for finding out all the Pareto Optimal solutions***

III. CONCLUSION

This paper explores the information from various resources like scholarly works, conference presentation and personal conversation. Finally, the area of Multi-Objective Optimization Problems is extremely complex and mathematically difficult, with many under-researched areas and outstanding problems. The application of Multi-Objective Optimization is challenging and increasing day by day so finally, with regard to future perspective, it requires more research, design and development on Multi-Objective Optimization Algorithms.

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